SRR & CVR GOVT. DEGREE COLLEGE

(Autonomous) NAAC 'B$^+$' Grade

DEPARTMENT OF MATHEMATICS

III B.Sc. MATHEMATICS

SEMESTER-V/ VI, PAPER – VII A

MATHEMATICAL SPECIAL FUNCTIONS

MODEL QUESTION PAPER

Time: 3Hrs **Max.Marks: 60**

SECTION-A

Answer any FIVE questions **5x4=20M**

1. Evaluate $\int_0^a x^4\sqrt{a^2-x^2}$ dx.
2. Prove that $\int_0^1 x^m(\log x)^n dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$.
3. Show that $L_2(x) = \frac{1}{2!}(2-4x+x^2)$.
4. Show that $L_n(x) = \frac{e^x}{n!}\frac{d^n(x^n e^{-x})}{dx^n}$.
5. Find Hermite Polynomials for n=0, 1, 2.
6. Evaluate $\int_{-\infty}^{\infty} xe^{-x^2} H_n(x).H_m(x)dx$.
7. Prove that $P_3(x) = \frac{1}{2}(5x^3-3x)$.
8. Show that $P_n(x)$ is the coefficient of h^n in the expansion in ascending powers of $(1-2xh+h^2)^{-1/2}$.
9. Prove that $J_{-n}(x) = (-1)^n J_n(x)$.
10. Prove that $\frac{d}{dx}[x^{-n}J_n(x)] = -x^{-n}J_{n+1}(x)$.

SECTION – B

Answer ALL questions **5x8=40 M**

11. (a) When n is a positive integer, prove that $\Gamma\left(-n+\frac{1}{2}\right)=\frac{(-1)^n 2^n\sqrt{\pi}}{1.3.5.....(2n-1)}$.

(OR)

(b) Prove that $B(l,m)=\frac{\Gamma(l)\Gamma(m)}{\Gamma(l+m)}$.

12. (a) Prove that $\frac{1}{1-t}e^{-tx/(1-t)}=\sum_{n=0}^{\infty}t^n L_n(x)$

(OR)

(b) Prove that $xL_n''(x)+(1-x)L_n'(x)+nL_n(x)=0$.

13. (a) State and Prove Rodrigue's formula for $H_n(x)$.

(OR)

(b) State and Prove Orthogonal Properties of Hermite Polynomials.

14. (a) Prove that $(2n+1)xP_n=(n+1)P_{n+1}+nP_{n-1}$.

(OR)

(b) Show that $\int_{-1}^{1}P_m(x).P_n(x)\,dx=0\ if\ m\neq n$.

15. (a) Prove that $xJ_n'(x)=nJ_n(x)-xJ_{n+1}(x)$.

(OR)

(b) Show that $J_{1/2}(x)=\sqrt{\frac{2}{\pi x}}\sin x$.